Predation by Stock Price Manipulation

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Abstract

We develop a model in which feedback effects from equity markets allow uninformed traders to profit by short selling a firm's stock while going long on its product market competitor. As this strategy distorts the investment of the firm targeted by short selling to the benefit of its rival, we label it *predation by stock price manipulation.* A short selling ban does not prevent manipulation since the speculator can still induce a firm to underinvest by establishing a long position in its rival. Our analysis unveils how competitive interactions among firms expand the scope of manipulation, providing new insights into equity markets and short sales regulation.

Keywords: Stock price manipulation, predation, product market competition, feedback effect.

JEL classification: D43, D82, D84, G14, G31.

*We thank Nihad Aliyev (Discussant), Heitor Almeida, Dan Bernhardt, Murillo Campello, Gustavo Cortes, Igor Cunha, Itay Goldstein, Idan Hodor, Charlie Kahn, Stefan Krasa, Jiang Luo (Discussant), Evgeny Lyandres, Julio Riutort, Vladimir Vladimirov, as well as seminar participants at the University of Illinois Urbana-Champaign, Universidade Federal de Pernambuco, the 2019 Latin American Meeting of the Econometric Society, the 2019 Meeting of the Brazilian Econometric Society, the 2022 Vietnam Symposium in Banking and Finance, and the 2022 FMA Asia/Pacific for their comments and suggestions. All remaining errors are ours. The paper was previously circulated under the title "Predatory Stock Price Manipulation".
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1 Introduction

Although secondary equity markets do not directly affect capital flows to firms, they contain important information that may be helpful to managerial decision-making (Bond et al., 2012). This feedback channel suggests that speculators can trade equity strategically to induce particular outcomes even when uninformed about economic fundamentals. This is known as stock price manipulation (Allen and Gale, 1992). In the context of product markets, speculators can exploit competition between rivals to profit by trading multiple stocks in a way that benefits particular firms. While a rich body of literature investigates how traders can harness feedback effects, little is known about how product market interactions can expand the boundaries of stock price manipulation.

In a seminal article, Goldstein and Guembel (2008) show that uninformed speculators can profit by short selling a firm, inducing it to underinvest, thereby reducing its value. In their single-firm model, profitable manipulation is one-sided, as uninformed buy orders lead to overinvestment, which also results in losses to shareholders. Our paper shows how competitive interactions among firms fundamentally change the scope for manipulation, enabling a manipulative strategy composed of buy and sell orders, with novel implications for short selling regulations, equity and product markets.

We augment the model of Goldstein and Guembel (2008) and introduce rival duopolists that make investments that are strategic substitutes and have their stocks traded in the equity market. In equilibrium, an uninformed speculator short sells a firm's share while buying its competitor's share in a *predatory stock market order*.[1] This strategy leads to the unilateral cancellation of investments by the

[1] We refer to "predatory" in the sense of predation among competitors, as in Bolton and Scharfstein (1990). The term is not directly related to predatory trading that exploits other

firm targeted by short selling, transferring value to its competitor. Therefore, we refer to this trading strategy as *predation by stock price manipulation*. By pairing trades in opposite directions, the speculator "doubles down" on her bet that only one firm will invest, thus profiting in two ways: by closing the short and long positions at the lowest and highest possible prices, respectively.

Regarding empirical relevance, the pairing of buy and sell trades of stocks within the same industry is well documented (Barardehi et al., 2022). While this trading pattern is typically attributed to informed traders, it cannot be fully explained by factors such as information diffusion and liquidity provision, suggesting that other economic channels may be at play (Chen et al., 2019). Our theory suggests one such channel, namely feedback effects in competitive product markets.

In our model, two firms make simultaneous investment decisions that negatively affect each other's value. While one of the firms has a known profitable project and invests with certainty (the "safe" firm), the other firm faces uncertainty about its prospects, and its manager learns from equity prices to make a decision (the "risky" firm).[2] Following certain stock market outcomes, the manager of the risky firm optimally withholds the investment, which benefits the competitor. We show how the speculator can achieve this with our proposed strategy, even when uninformed about the risky firm's underlying economic fundamentals.

In our equilibrium, the speculator always plays a long-short strategy regardless of her information. Specifically, the speculator targets the risky firm with short selling when either negatively informed or uninformed about its project in an attempt to induce it to cancel the investment. If successful, the speculator drives the

traders' needs to increase or decrease their positions, as in Brunnermeier and Pedersen (2005) and Brunnermeier and Oehmke (2014).

[2]The assumption that firm insiders learn from equity prices has received support from both theory and empirical evidence. See Bond et al. (2012) for a survey.

firm's value to zero while making its competitor attain its highest value possible by investing alone, thus gaining on the long and short legs of the portfolio. Interestingly, if the speculator has positive information about the risky firm's investment, she buys that firm's stock while short selling its safe competitor's stock. The underlying rationale is that going long on the risky firm ensures its investment, which decreases the value of the competitor's investment, again generating profits in both positions. Thus, even with no feedback channel from the stock market to the safe firm, it can still be targeted by short selling due to strategic substitution.

Short selling restrictions must strike a balance between preventing value-destroying manipulation and allowing informed short selling that improves price discovery and prevents firms from undertaking projects with negative NPV (Engelberg et al., 2012; Deng et al., 2020). We leverage the features of our model to study manipulation under a short selling ban. In particular, our two-stock framework opens two channels of information flows from the speculator to firms and financial markets, suggesting that manipulation might not be restricted to short sales. In this scenario, we establish that going long on the competitor can suffice to induce the risky firm to underinvest, which an uninformed speculator can exploit and, again, profit from strategic substitution in investments. Hence, although short selling constraints can alleviate selling pressure directly, they do not prevent manipulation when competitive interactions are present.

Notably, a comparison between our equilibrium with short selling and the equilibrium under a short selling ban reveals that *both* firms lose value in the latter. While the safe firm loses value simply because its competitor is more likely to invest, the risky firm loses value because the shorting restriction affects both the negatively informed and uninformed speculator equally, which is value-destroying in ours and Goldstein and Guembel (2008)'s benchmark model. That buy-side manipulative pressure on competitors can be value-destroying for both firms relative to manipulation with short selling adds competitive considerations as another layer to the discussion on the effectiveness and desirability of short selling regulations.

Several papers investigate the real economic implications of various feedback channels (e.g., Jarrow (1992); Dow and Gorton (1997); Khanna and Sonti (2004); Goldman and Strobl (2013); Edmans et al. (2015); Dow et al. (2017)). In particular, Goldstein and Guembel (2008) lay the theoretical foundation for manipulation via short selling. More recently, Campello et al. (2024) study the impact of short selling costs and show how managers can use stock repurchase contracts to signal quality and prevent manipulation. Other papers investigate how information production may be useful for managerial decision-making (Chang and You, 2010; Gao and Liang, 2013; Goldstein and Yang, 2017, 2019; Terovitis, 2022). Of note, Yang and Xiong (2021) study endogenous information disclosure when oligopolists learn from asset prices in making production decisions. Finally, Terovitis and Vladimirov (2024) explore how buy-side manipulation can arise by attracting high-quality stakeholders to the target firm.

We contribute to the literature on feedback effects by highlighting how competitive interactions can alter the scope of manipulation. As practical implications, the prevalence of manipulation may not be associated solely with the amount of short interest outstanding, and pairs trading does not necessarily reflect information (Chen et al., 2019; Barardehi et al., 2022). More generally, we show how indirect informational channels through firm networks, such as learning from peers' valuations (Foucault and Fresard, 2014), may be subject to manipulation motives. Our results also suggest that the ability of equity markets to improve allocation via the informational content of prices depends not only on the prevalence of manipulation and managerial incentives (Goldstein and Guembel, 2005; Singh and Yerramilli, 2014; Strobl, 2014) but also on the nature of firms' interactions in product markets.

Our results add to the discussion surrounding controversial short selling regulations. Although short selling restrictions can prevent "bear raids" that adversely drive stock prices down (Campello and Graham, 2013; Brunnermeier and Oehmke, 2014), they can also undermine stock price informativeness and its disciplining effect (Karpoff and Lou, 2010; Fang et al., 2016; Engelberg et al., 2017; Gao et al., 2020; Matta et al., 2024). By relaxing the assumption that firms are isolated decision makers, our model shows how competitive interactions can translate into trading strategies, offering an appropriate framework to study possible limitations of stock market interventions. In particular, we unveil how manipulation is not restricted to short selling in firm networks, suggesting that competitive interactions can impose "leakages" to short selling regulations.

A rich body of work studies the interplay between product markets and capital structure (Brander and Lewis, 1986; Maksimovic, 1988; Harris and Raviv, 1991), insider trading (Peress, 2010), industry returns (Bustamante and Donangelo, 2017), and investment decisions (Bustamante and Fresard, 2020). Lyandres (2006) shows how debt as a commitment device for engaging in aggressive output strategies (Brander and Lewis, 1986) depends on the degree of competitive interaction between product market rivals. In addition, several papers study the strategic effects of financial interests on opponents, either in competition or in litigation. Hansen and Lott (1995) establish that an incumbent firm might take a short position in a potential entrant to facilitate post-entry predation. Tookes (2008) argues and provide empirical evidence that informed traders take positions in product market competitors, especially following events in large firms. Choi and Spier (2018) show how, by taking a short position in a defendant firm, a plaintiff credibly threatens to bring a suit to trial, leading to larger settlements. We show that the presence of financial feedback in product markets enables profitable positions in competitors even by uninformed speculators and without post-entry considerations. In particular, we identify manipulation as an alternative means of predation that allows traders to profit from transferring value across opponents.

The rest of the paper is organized as follows. Section 2 introduces our baseline model, largely based on Goldstein and Guembel (2008). Section 3 presents our main results with short selling and under a short selling ban. Section 4 concludes, and Appendix A contains the proofs of our results.

2 The Model

This section presents our baseline model of product market competitors' investment decisions and stock trading. There are four dates $t \in \{0, 1, 2, 3\}$ and two firms that make simultaneous investment decisions of interdependent values. In $t = 0$, a risk-neutral strategic speculator may learn private information about how the investment of one firm affects the other. Trading of both firms' stocks take place in the financial markets in $t = 1$ and $t = 2$. Finally, in $t = 3$ the firms make investment decisions, open stock positions are liquidated and payoffs are realized.

2.1 The Product Market

There are two firms, indexed by $i \in \{A, B\}$, that compete in a product market. Each firm is run by a manager who can make an investment $I_i \in \{0, K\}$ of gross profitability Θ per dollar invested. A firm's type θ_i^ω, $\omega \in \{l, h\}$, measures its exposure to a negative externality imposed by its competitor's investment. Firm i's investment project is "strong" when $\omega = l$ ($\theta_i^\omega = \theta^l$) and "weak" when $\omega = h$ ($\theta_i^\omega = \theta^h$), where $\theta^h > \theta^l > 0$. Types occur with equal probability. The net value of the investment by firm i depends on the firm's own project type and both firms' investment decisions as follows:

$$V_i(I_i, I_{-i}, \theta^\omega) = (\Theta - \theta_i^\omega I_{-i})\, I_i, \tag{1}$$

where $-i$ indexes firm i's competitor. Note that profits from investing depend on types only when both firms invest. Moreover, not investing is an outside option with a value of zero with certainty.[3] Thus, we can interpret $V_i(I_i, I_{-i}, \theta^\omega)$ as the NPV of firm i's investment, implying that investment decisions are strategic substitutes. In addition, we assume that

$$\theta^h > \frac{\Theta}{K} > \frac{\theta^h + \theta^l}{2}, \tag{2}$$

which means that a firm with a weak project has negative value when both firms invest, but *ex-ante* a firm should invest. Thus, a firm attains the highest possible value when it invests alone. If both firms invest, a strong firm still makes a profit, whereas a weak firm loses value.[4]

To simplify notation, we adapt our product market model to the more familiar notation of Goldstein and Guembel (2008). Specifically,

$$V^{++} = \Theta K,\ V^{+} = \left(\Theta - \theta^l K\right) K,\ V^{-} = \left(\Theta - \theta^h K\right) K.$$

Therefore, condition (2) now reads

$$\overline{V} > 0 > V^{-}, \tag{3}$$

[3]For ease of exposition, we omit the assets in place in our definition of firm value. Including them would have no major implications on the analysis.

[4]Throughout the paper, we use the term "strong firm" and "weak firm" firm to refer to a firm to with a strong and weak investment project, respectively.

where $\overline{V} = \frac{V^+ + V^-}{2}$ is the *ex-ante* expected value of investment when a firm's type is unknown. Finally, suppose that

$$2V^+ > V^{++}, \tag{4}$$

which means that the combined investment of two strong firms is more valuable that the investment of a single firm. However, because $V^{++} > V^+$, investment by a single firm is *ex-ante* better than the combined investment of firms of unknown type. The reason is that firms do not internalize the negative effect of their own investment on their competitors. As a result, both managers investing is the unique equilibrium in dominant strategies of the product market model when both types are unknown and no further information is generated.

Although simple, our product market model features the key element for our proposed stock trading strategy to arise: strategic substitution in investments. By introducing competitive interactions as an additional determinant of firms' values, we allow for a greater range of payoffs as compared to Goldstein and Guembel (2008)'s single-firm investment problem. In turn, the speculator internalizes these interactions in her actions, thereby significantly expanding her possible stock trading strategies. As a result, equilibria in a full-fledged stock trading model where the types of both firms are unknown are prohibitively large to characterize.

To simplify the equilibrium's characterization, we assume that it is public knowledge that firm B is strong: $\theta_B^\omega = \theta^l$. Hence, firm B is "safe", its manager will certainly invest, and the value of the investment is solely determined by whether firm A also invests or not. Thus, V_B will be either V^{++} or V^+ when firm A does not invest or invests, respectively. In contrast, firm A has a risky investment, and V_A will be either V^+, V^-, or 0, depending on its type and investment decision. This assumption substantially decreases possible strategies to consider while keeping the essential elements for our proposed long-short trading strategy.

2.2 The Stock Market

Stock trading in the financial market closely follows Goldstein and Guembel (2008), except that there are two stocks that can be traded. In $t = 0$ the speculator receives an informative signal S about firm A's type with probability α. Otherwise, she receives an uninformative, empty signal $\emptyset$. By assuming that firm A is strong or weak with probability $\frac{1}{2}$ each, the speculator's possible signals are θ^l, θ^h, and $\emptyset$ with respective probabilities $\frac{\alpha}{2}$, $\frac{\alpha}{2}$, and $1 - \alpha$, which we refer to as "positively informed", "negatively informed", and "uninformed" speculator.[5] At each trading period $t = 1, 2$, the speculator submits market orders to trade the stock of each firm, i.e., orders that are not contingent on current stock prices. We define $u_{t,i}$ as the speculator's order to buy, sell, or not trade the stock of firm i at time t.

Besides the strategic speculator, there are two more types of agents participating on the stock market: noise traders and market makers. The noise traders submit market orders $\eta_{t,i}$ to buy, sell, or not trade the stock of each firm in each period with equal probabilities. We assume that the noise traders' order are uncorrelated both serially and across firms. We denote -1, 0, and 1 as a market order to sell, not trade, and buy the stock of each firm in each period, respectively. Therefore, both $u_{t,i}$ and $\eta_{t,i}$ take values in $\{-1, 0, 1\}$ for $i = A, B$ and $t = 1, 2$. When an agent does not trade a stock, we sometimes say they submit a "null order" of size zero.

[5] Note that the term "positively informed"speculator refers to the "low" value of the signal $S = \theta^l$, which means that the firm is strong. Not to be confused with Goldstein and Guembel (2008) where the term refers to the high value of the signal h. The same applies to the "negatively informed" speculator case.

Following Kyle (1985), there is a market maker who sets equity prices and executes order flows out of his inventory. We make the standard assumption that the market maker is uninformed about firm A's type. In each round of trading, the orders received are processed at prices equal to the expected value of firms conditional on the information available up to that period. For each period and firm, the market maker observes only the total order flow $Q_{t,i} = u_{t,i} + \eta_{t,i}$, but not its individual components. Therefore, $Q_{t,i} \in \{-2, -1, 0, 1, 2\}$ for each stock and trading period. The market maker sets stock prices equal to the expected value of the firm given the order flows of both stocks, i.e., $p_{i,1}(Q_{1,A}, Q_{1,B}) = \mathbb{E}(V_i|Q_{1,A}, Q_{1,B})$, and $p_{i,2}(Q_{1,A}, Q_{2,A}, Q_{1,B}, Q_{2,B}) = \mathbb{E}(V_i|Q_{1,A}, Q_{2,A}, Q_{1,B}, Q_{2,B})$.

In this framework, the speculator's trading strategy can be defined as follows. In $t = 1$, she submits market orders on both stocks based only her signal S. In $t = 2$, her market orders are based on her signal, her orders and the total order flows in $t = 1$.[6]

As we mentioned in Section 2.1, assuming that firm B's investment is strong brings about an important feature that simplifies the stock trading analysis. Because firm B's manager invests with certainty, there is no feedback effect from the stock market to firm B's investment. However, the value of firm B is still determined by the stock market via firm A's investment. Therefore, in the presence of feedback effects via manager A learning from stock prices, the speculator can still profit from her position in B by affecting the investment of firm A. This profit, however, comes solely from the position established in $t = 1$. The reason is that, once the market maker observes $Q_{2,A}$ and $Q_{2,B}$, the uncertainty about whether firm A invests or not is resolved, and $p_{2,B}$ is set to the true value of firm B's invest-

[6] For notation simplicity, we follow Goldstein and Guembel (2008) and make trading strategies contingent on order flows only instead of stock prices.

ment. Therefore, any order $u_{2,B}$ is opened and closed at equal prices, and closing a position established in $t = 1$ is the same as holding it to liquidation. As we will see in detail in Section 3 and Appendix A, this property substantially reduces contingencies and possible deviations when analyzing equilibrium strategies.

3 Predation by Stock Price Manipulation

Suppose that the manager of firm A is uninformed about the profitability of his firm's investment opportunity. As such, he uses the information coming from stock prices to guide his investment decision, thus opening a feedback channel from the financial market to firm A's real investment. Therefore, as in standard feedback models, an equilibrium must endogenously determine stock prices and investment decisions. In our model, trading strategies and price setting must account for the two firms, but only the investment decision of firm A's manager depends on stock prices. Therefore, a Perfect Bayesian Nash Equilibrium of our model consists of the following:

i) A trading strategy by the speculator $\{u_{1,i}(S), u_{2,i}(S, u_{1,i}, Q_{1,A}, Q_{1,B})\}, i = A, B$ that maximizes her expected final payoff given price-setting by the market maker, managers' investment decision, and the information available in each trading period.

ii) An investment decision by firm A's manager that maximizes expected firm value given the speculator's trading strategy and price-setting by the market maker.

iii) A price-setting rule by the market maker $p_{i,1}(Q_{1,A}, Q_{1,B})$ and $p_{i,2}(Q_{1,A}, Q_{2,A}, Q_{1,B}, Q_{2,B}), i = A, B$ such that the market maker breaks even in expectation given trading strategies and investment decisions.

As in Goldstein and Guembel (2008)'s main result, low stock prices of firm A indicate that the speculator might have negative information about the firm, leading firm A's manager to optimally call off the investment. Therefore, an uninformed speculator can profit from short selling firm A by inducing it to not invest, driving its value to zero and closing the short position at a profit. As standard in the literature, we refer to an uninformed trading strategy that profits from feedback effects as manipulation.

To ensure the existence of profitable manipulation, we make the following standard assumption:

$$\frac{\alpha}{2}V^{-} + (1-\alpha)\overline{V} < 0, \tag{5}$$

which states that the probability of an informative signal is high enough that firm A's manager optimally calls off the investment if it is revealed that the speculator is not positively informed.

3.1 Manipulation With Short Selling

In Goldstein and Guembel (2008), manipulation consists on the uninformed, profitable short selling of a firm in equilibrium. Here, however, there is one more layer to consider. If the speculator is successful in leading firm A to underinvest, she can also profit from a long position in firm B. Even if uninformed, the speculator still has the informational advantage of knowing she will continue to short sell firm A in $t = 2$. Therefore, in $t = 1$, she can establish a long position in firm B when there is still uncertainty about firm A's investment, and close the position when firm B attains the highest possible value V^{++}. Because a long-short strategy can generate profit on both legs of the portfolio by transferring value from the firm targeted by the short order when it cancels investments to the one on the long position, we refer to it as a *predatory order*. Our main result establishes that an uninformed predatory order against the risky firm exists in equilibrium.

Consider the following trading strategy, which we denote $\tilde{u}$. First, we have that

$$\tilde{u}_{1,A}(S = \theta^l) = 1,\ \tilde{u}_{1,B}(S = \theta^l) = -1,$$

which means that the positively informed speculator establishes a long position in firm A and a short position in firm B. In $t = 2$, the positively informed speculator trades as follows.

$$\tilde{u}_{2,A}(S = \theta^l, Q_{1,A}, Q_{1,B}) = \begin{cases} 1 & \text{if } Q_{1,A} \in \{0, 1\} \text{ and } Q_{1,B} = 0 \\ -1, 0, \text{ or } 1 & \text{if } Q_{1,A} = 2 \text{ or } Q_{1,B} \in \{-1, -2\}, \end{cases}$$

and

$$\tilde{u}_{2,B}(S = \theta^l, Q_{1,A}, Q_{1,B}) = -1, 0, \text{ or } 1, \text{ for any } Q_{1,A}, Q_{1,B}.$$

Both the negatively informed and the uninformed speculator use the same trading strategy as follows:

$$\tilde{u}_{1,A}(S \in \{\theta^h, \emptyset\}) = \begin{cases} 0 & \text{with probability } \mu \\ -1 & \text{with probability } 1 - \mu, \end{cases}$$

with μ endogenously determined in equilibrium. Trading of firm B's stock is

$$\tilde{u}_{1,B}(S \in \{\theta^h, \emptyset\}) = 1.$$

In $t = 2$, the trading strategies are

$$\tilde{u}_{2,A}(S \in \{\theta^h, \emptyset\}), Q_{1,A}, Q_{1,B}) = \begin{cases} -1 & \text{if } Q_{1,A} \in \{0,1\}, \text{ and } Q_{1,B} = 0 \\ -1, 0, \text{ or } 1 & \text{if } Q_{1,A} \in \{-2,-1\} \text{ or } Q_{1,B} \in \{1,2\}, \end{cases}$$

and

$$\tilde{u}_{2,B}(S \in \{\theta^h, \emptyset\}, Q_{1,A}, Q_{1,B}) = -1, 0, \text{ or } 1, \text{ for any } Q_{1,A}, Q_{1,B}.$$

In words, if $S = \theta^l$ and the order flows in $t = 1$ do not reveal that the speculator is positively informed, she further buys the stock of firm A. Since the only instance in which the speculator goes long on firm A and short on firm B is when $S = \theta^l$, if $Q_{1,A} = 2$ or $Q_{1,B} < 0$, inclusive, the speculator's signal is revealed to the market maker in $t = 1$. In this case, $p_{1,A} = p_{1,B} = V^+$, both firms invest, and the speculator does not profit from any further transactions, thus randomizing between buy, sell and null orders in $t = 2$. Interestingly, the positively informed speculator short sells the stock of the strong firm. The reason this is profitable is that, by going long on firm A, the speculator ensures that it will invest, and hence firm B will attain its lowest possible value of V^+. Therefore, if the order flows do not reveal the speculator's information in $t = 1$, she is able to establish a short position in firm B at a price that is higher than V^+ because the market maker still assigns a positive probability for firm A not investing. In this case, the speculator has the informational advantage of knowing she will further go long on firm A, ensuring its investment.

As in Goldstein and Guembel (2008), the negatively informed and uninformed speculators randomize between selling and not trading firm A in $t = 1$. The randomization parameter μ is the same for both types of speculators, as they face the same trade-off in $t = 1$. In this case, the speculator goes long on firm B, which is profitable because she attempts to make firm A cancel the investment. If successful, she establishes a long position in B and closes it at the highest value V^{++}. As the uninformed speculator places a predatory order against a firm of unknown type for the benefit of its competitor, we refer to this strategy as predation by stock price manipulation.

There is one distinction between the strategies described here and those in Goldstein and Guembel (2008)'s original one-firm model. Here, the negatively informed and uninformed speculators play an identical strategy in both trading periods. In Goldstein and Guembel (2008), the uninformed speculator who does not trade in $t = 1$ also does not trade in $t = 2$ when $Q_1 = 0$. In that case, the speculator is indifferent between selling and not trading, with not trading composing the equilibrium. Here, however, this is not the case due to the long position in firm B. By selling firm A in $t = 2$, the uninformed speculator increases the probability that firm A will not invest, which increases her expected returns on firm B. As a result, the uninformed speculator strictly prefers to short sell firm A in $t = 2$ even if she did not trade in $t = 1$ and $Q_{1,A} = Q_{1,B} = 0$.

Two more features of $\tilde{u}$ and the stock market are worth noting. First, the order flows of *both* stocks can reveal information to the market maker. In particular, because the speculator always trades the stock of firm B, any order flow $Q_{1,B} \neq 0$ reveals her trading in a way that prevents her from profiting. Therefore, $Q_{1,B} = 0$ is a necessary condition for any speculator to profit, and following $Q_{1,B} \neq 0$ we assume that the speculator randomizes between buying, selling and not trading firm A. Finally, $u_{2,B}$ and $Q_{2,B}$ are irrelevant for the speculator's profits. As we discuss in Section 2.2, all that matters for the value of firm B is whether firm A invests or not. Once $Q_{2,A}$ is observed, this uncertainty is resolved, which sets $p_{2,B}$ to the firm's true value, making any trade of firm B's stock in $t = 2$ of net zero value. Therefore, we assume that the speculator randomizes among the possible actions on firm B with strictly positive probabilities.

Our main result establishes the existence of an equilibrium with predation by stock price manipulation.

Proposition 1 *Suppose that conditions (3), (4), and (5) are satisfied. If the stock market is sufficiently informative, there exists an equilibrium where the speculator trades according to* $\tilde{u}$*.*

In words, if α is sufficiently large, there exists an equilibrium with predation by manipulation. In it, the speculator always use a long-short strategy regardless of her signal. The negatively informed and uninformed speculators use the same trading strategy, which we define in detail in the following corollary that follows from the proof of Proposition 1.

Corollary 1 *In the equilibrium where the speculator trades according to* $\tilde{u}$*, both the negatively informed and the uninformed speculator short sell firm* A *in* $t = 1$ *with probability* $1 - \mu$*, where*

$$\mu = \begin{cases} 0, & \text{if } 4V^{++} \leq (3+2\alpha)V^{+} + V^{-} \\ \dfrac{4V^{++} - (3+2\alpha)V^{+} - V^{-}}{(5-2\alpha)V^{+} - V^{-}}, & \text{if } 4V^{++} > (3+2\alpha)V^{+} + V^{-}. \end{cases} \tag{6}$$

Note that $\mu < 1$ by condition (4), meaning that the negatively informed and uninformed speculators either sell stock A in $t = 1$ with certainty or mix between selling and not trading. In addition, note that μ is decreasing in α, increasing in $\frac{V^{++}}{V^{+}}$, and decreasing in $\frac{V^{-}}{V^{+}}$. These comparative statics generate empirical predictions of the product and financial market aspects that drive short selling in risky firms that are exposed to feedback effects, that is, conditional on $S \in \{\theta^{h}, \emptyset\}$.

First, short selling firm A is more likely when the stock market is more informative. Second, short selling is less likely when the value of a single firm that invests is large relative to the value of a strong firm when both invest. The reason is that, when V^{++} is higher relative to V^{+}, the profit from closing the long position in firm B increases relative to the profit from closing short positions in A. Since the only way the speculator can profit from stock B is with the position established in $t = 1$, the speculator makes her $t = 1$ orders less likely to be revealed by not trading firm A. Finally, more short selling is expected when the losses from bad projects are smaller relative to good projects when both firms invest. In such case, short positions in firm A are established at higher prices, especially in $t = 1$, thus increasing the probability of initial short selling.

To further illustrate our results, we provide two examples of parameters that compose equilibria with trading strategy $\tilde{u}$. The values $V^{+} = 10$, $V^{-} = -2$, $V^{++} = 11$, and $\alpha > \frac{4}{5}$ constitute an equilibrium where the negatively informed and uninformed speculators always short sell firm A in $t = 1$ ($\mu = 0$). The values $V^{+} = 5$, $V^{-} = -4$, $V^{++} = 6$, and $\alpha = \frac{1}{2}$ form an equilibrium where the negatively informed and uninformed speculators mix between not trading and short selling firm A in $t = 1$ ($\mu = \frac{1}{3}$).

Our main result underscores how strategic substitution in investment decisions introduces a new source of profits for the speculator. Either by ensuring that firm A invests when positively informed or by attempting to make firm A call off investments when negatively informed or uninformed, the speculator always establishes a long-short position. By internalizing competitive interactions into her trading strategy, the speculator is able profit on both legs of the portfolio, in expectation.

3.2 Manipulation Under a Short Selling Ban

The possibility of trading two stocks opens two avenues of information from the speculator to firm A's manager and the market maker. Therefore, under this framework, it is not clear whether interventions that aim to curb short selling are effective against manipulative trading. Even if the speculator cannot establish a short position in a firm, she can still convey negative information with a long position in its competitor. In addition, although short selling enables value-destroying manipulation in the baseline model by Goldstein and Guembel (2008), it also improves price discovery and leads to better investment decisions when employed by informed traders. Therefore, it is disputable under which conditions short selling constraints are desirable (Gao et al., 2020; Aliyev and Deng, 2024). In this section, we leverage the unique features of our two-stock model to study the effects of a short selling ban on both manipulative trading and firm value.

3.2.1 Manipulation With Long Positions

Assume the same financial market structure described in Section (2.2), except that the speculator cannot place a short sale order. That is, the speculator can only sell stock that she holds. Since the speculator starts with no position in either firm, she can only buy or not trade the stock of each firm in $t = 1$. Under this restriction, consider the following trading strategy, which we denote u^{sb}:

$$u^{sb}_{1,A}(S = \theta^l) = 1, \; u^{sb}_{1,B}(S = \theta^l) = 0,$$

which means that the positively informed speculator places a long and a null order on firms A and B, respectively. In $t = 2$, the positively informed speculator trades as follows:

$$u^{sb}_{2,A}(S=\theta^l, Q_{1,A}, Q_{1,B}) = \begin{cases} 1 & \text{if } Q_{1,A} \in \{0,1\} \text{ and } Q_{1,B} \in \{0,1\} \\ -1, 0, \text{ or } 1 & \text{if } Q_{1,A} = 2 \text{ or } Q_{1,B} = -1, \end{cases}$$

and

$$u^{sb}_{2,B}(S=\theta^l, Q_{1,A}, Q_{1,B}) = -1, 0, \text{ or } 1 \text{ for any } Q_{1,A}, Q_{1,B}.$$

Again, the negatively informed and the uninformed speculators use the same trading strategy, as follows:

$$u^{sb}_{1,A}(S \in \{\theta^h, \emptyset\}) = 0,\ u^{sb}_{1,B}(S \in \{\theta^h, \emptyset\}) = 1,$$

and $t = 2$ trading strategies are

$$u^{sb}_{2,A}(S \in \{\theta^h, \emptyset\}, Q_{1,A}, Q_{1,B}) = \begin{cases} 0 & \text{if } Q_{1,A} \in \{0,1\} \text{ and } Q_{1,B} \in \{0,1\} \\ -1, 0, \text{ or } 1 & \text{if } Q_{1,A} - 1 \text{ or } Q_{1,B} = 2, \end{cases}$$

and

$$u^{sb}_{2,B}(S \in \{\theta^h, \emptyset\}, Q_{1,A}, Q_{1,B}) = -1, 0, \text{ or } 1 \text{ for any } Q_{1,A}, Q_{1,B}.$$

In words, the speculator does not trade a share when she would short sell it in $t = 1$ when following $\tilde{u}$. In $t = 2$, she repeats her $t = 1$ trading of firm A unless her position was revealed by the order flows, in which case she randomizes across all possible actions. Again, her $t = 2$ trading of firm B is innocuous so we assume she randomizes across actions regardless of her signal and past order flows.

Our next result establishes the conditions for which u^{sb} constitutes an equilibrium with a short sale ban.

Proposition 2 *Suppose that conditions (3), (4), and (5) are satisfied and short selling is not allowed. If it holds that*

$$(3+\alpha)V^{+} + V^{-} \geq 2V^{++}, \tag{7}$$

there exists an equilibrium where the speculator trades according to u^{sb}.

With competition in strategic substitutes, a short selling ban does not prevent manipulation, as an uninformed speculator can still induce firm A underinvest. This transfers value from firm A to firm B, allowing the speculator to close the long position in B at a profit. With no short sales, buying firm B is how a negatively informed speculator profits from conveying negative information about firm A, which the uninformed speculator mimics. Both speculators make the same *ex-ante* expected profits by following this strategy, as the profit comes solely from the long position in firm B, therefore not depending on firm A's type. Finally, note that our parametric examples of equilibria with short selling in Section (3.1) also permit an equilibrium under a short sale ban as in Proposition 2.

Although a short selling ban does not completely prevent manipulation, in our equilibrium on Proposition 2, firm A is less likely to cancel investments than in the equilibrium in Proposition 1. However, since in both equilibria the negatively informed and uninformed speculator play the same trading strategy, a short selling ban hinders both manipulation and informed short selling in the same way, which leads us to our next result.

Corollary 2 *Let* V_i^{ss} *and* V_i^{sb} *be the values of firm* i *under the equilibrium in Proposition 1 and Proposition 2, respectively. The following holds*

(i). $\mathbb{E}(V_A^{ss}) > \mathbb{E}(V_A^{sb})$, *and* $\mathbb{E}(V_B^{ss}) > \mathbb{E}(V_B^{sb})$.

(ii). $\mathbb{E}(V_A^{ss}) - \mathbb{E}(V_A^{sb})$ *is increasing in* α *and* $\mathbb{E}(V_B^{ss}) - \mathbb{E}(V_B^{sb})$ *is decreasing in* α*.*

In words, the *ex-ante* expected values of both firms are lower in our equilibrium under a short selling ban relative to the equilibrium with short sales of Section (3.1). Additionally, when the financial market is more informative, this difference increases for firm A, but decreases for firm B.

Since in our equilibrium under a short selling ban firm A is more likely to invest, the *ex-ante* value of firm B obviously decreases. However, because the short sale ban affects the negatively informed and the uninformed speculator similarly, the value of firm A also decreases, by assumption (5). Moreover, when the stock market is more informative, it is more likely that underinvestment by firm A reflects negative information, so establishing a short sale ban allows more investment by firm A when it is weak, further destroying its value. As for firm B, all that matters is whether firm A invests or not. With short selling allowed, investment of firm A is more sensitive to α than under a short selling ban. Therefore, at high values of α, establishing a short selling ban increases investment by firm A, but not so much, which alleviates firm B's loss in expected value.

3.2.2 The Impossibility of Manipulation With Regular Sales

Condition (7) states that V^{++} cannot be too large, otherwise the speculator has an incentive to deviate from u^{sb}. Specifically, the speculator can further increase the probability that firm A will not invest by buying both firms in $t = 1$ and selling firm A in $t = 2$. In such deviation, the speculator closes the long position in firm A at a loss, but increases the expected value of the position in firm B. Condition (7) states that the gain on firm B is small relative to loss on firm A, therefore ensuring the optimality of u^{sb}. However, it raises the question of whether an equilibrium where the speculator sells firm A in $t = 2$ exists, as we discuss in our next result.

Proposition 3 *Suppose that conditions (3), (4), and (5) are satisfied and short selling is not allowed. Suppose that the positively informed speculator trades according to* u^{sb}*. If the negatively informed and uninformed speculators buy both firms in* $t = 1$ *and sell firm* A *in* $t = 2$*, they make a negative profit.*

Proposition 3 states that no equilibrium with a "regular" sale of stock A exists under our parametric conditions and a short selling ban. When the speculator holds long positions in both stocks and sells A in $t = 2$, inducing it to not invest, the speculator makes a profit on B, but also liquidates the long position in A at a value of zero. Condition (4) implies that the loss on A more than offsets the gain in B, so this strategy cannot compose an equilibrium.

In our framework with competing firms a short selling ban alleviates overall selling pressure when speculators are uninformed, but it does not prevent manipulation. On the one hand, speculators cannot circumvent a short sale ban with regular selling. However, they can still manipulate stock prices with buying pressure on competitors, highlighting how competitive interactions can expand the scope of manipulation and pose additional challenges to short selling interventions.

4 Concluding Remarks

We show how product market competition can shape stock price manipulation in the presence of feedback effects from secondary equity markets to firms' real decisions. In our model, uninformed speculators can exploit strategic substitution in competitors' investment levels by pairing a short sell and a buy trade on their stocks. We labeled this *predation by stock price manipulation* as it induces the firm targeted by short selling to forego profitable investments and lose value to its rival, allowing the speculator to close both positions at a profit.

Under a short selling ban, the speculator can still induce a firm to cancel investments by establishing a long position in its rival. In this scenario, the expected value of *both* firms in our model is lower when compared to our benchmark case with short sales. Thus, we show how encompassing competitive interactions in the presence of feedback effects changes the scope of manipulation, possibly undermining the effectiveness of short selling restrictions aimed at curbing manipulative trading.

Our focus on the interplay between manipulation and competitive aspects brings together the literature on feedback effects and predation. Since firms are not isolated decision makers and may base their decisions on information stemming from the stock market as a whole, we take an important step towards a better understanding of how product and financial markets are intertwined. A thorough comprehension of this relationship can aid regulators in designing policies to prevent manipulative trading that benefits a few firms at the expense of competitors while preserving the allocational role of stock prices in complex firm networks.

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Appendix A Proofs

Proof of Proposition 1

We follow Goldstein and Guembel (2008) and show that, given (3)—(5), $\tilde{u}$ is an equilibrium in four parts. In part A, we determine stock prices given both firms' investment decisions and the speculator's trading strategy. In part B, we show that $t = 2$ trading strategies are optimal given market prices, investment decisions, and $t = 1$ trading strategies. Part C shows that $t = 1$ trading strategies are optimal. Finally, part D discusses off-path outcomes.

A. Stock prices and firms' investments: First, firm B always invests because it is surely strong. Thus, any uncertainty about firm B's value comes solely from the investment of firm A. In contrast, firm A does not invest when its expected value is negative, which, by assumption (5), happens when order flows reveal that the speculator did not observe $S = \theta^l$.

In our framework of two stocks being traded, the order flow of both stocks can reveal information about the speculator's signal. However, a few features of the model and $\tilde{u}$ simplify characterization. As discussed in Section 3, the speculator cannot profit from a position established in firm B in $t = 2$, as any action at that point yields zero profits. In this case, we assume that the speculator randomizes across all possible actions $u_{2,B}$ regardless of her signal so we do not depend on off-path beliefs about $Q_{2,B}$. As a result, $Q_{2,B}$ is uninformative and can be disregarded in the characterization. In addition, any $Q_{1,B} < 0$ reveals that $S = \theta^l$ whereas $Q_{1,B} > 0$ reveals that $S \in \{\theta^h, \emptyset\}$, making the expected value of firm A's investment negative.[7] Therefore, any $Q_{2,A} \neq 0$ prevents the speculator from profiting from the positions at either firm. Using Bayes rule, the prices along the equilibrium path are:

$$\begin{aligned}&p_{2,A}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = 0,\ p_{2,B}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = V^{++},\ \text{when } Q_{1,A} \in \{-2, -1\} \text{ and}\\ &Q_{1,B} \in \{0, 1, 2\};\ \text{or when } Q_{1,A} \in \{0, 1\} \text{ and either } Q_{1,B} \in \{1, 2\} \text{ or } Q_{2,A} \in \{-2, -1\},\end{aligned} \tag{8}$$

$$\begin{aligned}&p_{2,A}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = p_{2,B}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = V^{+},\ \text{when } Q_{1,A} = 2 \text{ and}\\ &Q_{1,B} \in \{-2, -1, 0\};\ \text{or when } Q_{1,A} \in \{0, 1\} \text{ and either } Q_{1,B} \in \{-2, -1\} \text{ or } Q_{2,A} \in \{1, 2\},\end{aligned} \tag{9}$$

$$p_{2,A}(0, 0, 0) = \overline{V},\ p_{2,B}(0, 0, 0) = V^{+}, \tag{10}$$

$$p_{2,A}(1, 0, 0) = \frac{\alpha V^{+} + \alpha\mu V^{-} + 2(1-\alpha)\mu\overline{V}}{\alpha(1+\mu) + 2(1-\alpha)\mu},\ p_{2,B}(1, 0, 0) = V^{+}. \tag{11}$$

Prices in $t = 1$ are computed as in Goldstein and Guembel (2008) and are given by

$$\begin{aligned}&p_{1,A}(Q_{1,A}, Q_{1,B}) = 0,\ p_{1,B}(Q_{1,A}, Q_{1,B}) = V^{++},\ \text{when } Q_{1,A} \in \{-2, -1\} \text{ and}\\ &Q_{1,B} \in \{0, 1, 2\};\ \text{or when } Q_{1,A} = 0 \text{ and } Q_{1,B} \in \{1, 2\},\end{aligned} \tag{12}$$

$$\begin{aligned}&p_{1,A}(Q_{1,A}, Q_{1,B}) = p_{1,B}(Q_{1,A}, Q_{1,B}) = V^{+},\ \text{when } Q_{1,A} = 2 \text{ and } Q_{1,B} \in \{-2, -1, 0\};\\ &\text{or when } Q_{1,A} \in \{0, 1\} \text{ and } Q_{1,B} \in \{-2, -1\},\end{aligned} \tag{13}$$

$$\begin{aligned}p_{1,A}(1, 0) &= \frac{1}{3}p_{2,A}(1, 0, 0) + \frac{2}{3}\frac{\alpha V^{+}}{\alpha(1+\mu) + 2(1-\alpha)},\\ p_{1,B}(1, 0) &= \frac{1}{3}\frac{\alpha(3+\mu) + 2(1-\alpha)\mu}{\alpha(1+\mu) + 2(1-\alpha)\mu}V^{+} + \frac{2}{3}\frac{\alpha\mu + 2(1-\alpha)\mu}{\alpha(1+\mu) + 2(1-\alpha)\mu}V^{++},\end{aligned} \tag{14}$$

[7] Later in the proof we discuss off-path cases where $Q_{1,A}$ and $Q_{1,B}$ are conflicting with respect to revealing the speculator's signal.

where $p_{2,A}(1,0,0)$ is given in (11). Finally,

$$\begin{aligned} p_{1,A}(0,0) &= \frac{1}{3}\overline{V} + \frac{\alpha}{3}V^{+}, \\ p_{1,B}(0,0) &= \frac{1+\alpha}{3}V^{+} + \frac{2-\alpha}{3}V^{++}. \end{aligned} \tag{15}$$

B. Trading strategies in $t=2$: Next, we show that there are no profitable deviations from $\tilde{u}$ in $t=2$ given the prices above. Recall that $u_{2,B}$ is innocuous, so we only have to account for possible deviations in $u_{2,A}$.

The positively informed speculator goes long on firm A and short on firm B in $t=1$. If $Q_{1,A}=2$ or $Q_{1,B}\in\{-2,-1\}$ inclusive, all actions generate zero profits on the trade of both stocks, so she is indifferent. Suppose that $Q_{1,B}=0$. If $Q_{1,A}=1$, buying firm A again yields a profit of $\frac{4}{3}V^{+}-p_{1,A}(1,0)-\frac{1}{3}p_{2,A}(1,0,0)$ on firm A's position, and $p_{1,B}(1,0)-V^{+}$ on firm B. Not trading firm A generates $\frac{2}{3}V^{+}-p_{1,A}(1,0)$ on firm A, and $p_{1,B}(1,0)-\frac{2}{3}V^{+}-\frac{1}{3}V^{++}$ on firm B. Selling generates $\frac{1}{3}p_{2,A}(1,0,0)-p_{1,A}(1,0)$ and $p_{1,B}(1,0)-\frac{1}{3}V^{+}-\frac{2}{3}V^{++}$. Clearly, buying again in $t=2$ dominates in both the long and short legs of the portfolio. If $Q_{1,A}=0$, buying firm A again generates a profit of $\frac{4}{3}V^{+}-p_{1,A}(0,0)-\frac{1}{3}\overline{V}$ on firm A, and $p_{1,B}(0,0)-V^{+}$ on firm B. Not trading yields $\frac{2}{3}V^{+}-p_{1,A}(0,0)$ and $p_{1,B}(0,0)-\frac{2}{3}V^{+}-\frac{1}{3}V^{++}$. Selling gives $\frac{1}{3}\overline{V}-p_{1,A}(0,0)$ and $p_{1,B}(0,0)-\frac{1}{3}V^{+}-\frac{2}{3}V^{++}$. Again, buying firm A yields higher profits on both legs, so the speculator buys in $t=2$.

The negatively informed speculator that short sells firm A and goes long on firm B: If $Q_{1,A}\in\{-2,-1\}$ or $Q_{1,B}\in\{1,2\}$, all actions generate zero profits, so she is indifferent. Suppose that $Q_{1,A}=Q_{1,B}=0$. Buying firm A yields $p_{1,A}(0,0)-\frac{2}{3}V^{+}-\frac{1}{3}\overline{V}$ on stock A, and $-p_{1,B}(0,0)+V^{+}$ on firm B. Not trading generates $p_{1,A}(0,0)-\frac{2}{3}V^{-}$ and $-p_{1,B}(0,0)+\frac{2}{3}V^{+}+\frac{1}{3}V^{++}$. Selling again gives $p_{1,A}(0,0)+\frac{1}{3}\overline{V}-\frac{2}{3}V^{-}$ and $-p_{1,B}(0,0)+\frac{1}{3}V^{+}+\frac{2}{3}V^{++}$. Selling stock A generates higher profits on both positions, so the speculator unambiguously chooses to sell.

The negatively informed speculator that does not trade stock A and goes long on firm B: If $Q_{1,A}=-1$ or $Q_{1,B}\in\{1,2\}$, all actions generate zero profits, so she is indifferent. Suppose that $Q_{1,B}=0$. If $Q_{1,A}=1$, buying firm A generates $V^{-}-\frac{2}{3}V^{+}-\frac{1}{3}p_{2,A}(1,0,0)$ on stock A, and $-p_{1,B}(1,0)+V^{+}$ on firm B. Not trading generates 0 on firm A, and $-p_{1,B}(1,0)+\frac{2}{3}V^{+}+\frac{1}{3}V^{++}$ on stock B. Selling gives $\frac{1}{3}p_{2,A}(1,0,0)-\frac{1}{3}\overline{V}$ on stock A, and $-p_{1,B}(1,0)+\frac{1}{3}V^{+}+\frac{2}{3}V^{++}$ on stock B, so she chooses to sell. If $Q_{1,A}=0$, buying firm A generates $V^{-}-\frac{2}{3}V^{+}-\frac{1}{3}\overline{V}$ on stock A, and

$-p_{1,B}(0,0) + V^{+}$ on stock B. Not trading yields 0 on stock A, and $-p_{1,B}(0,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$ on stock B. Selling yields $\frac{1}{3}\overline{V} - \frac{1}{3}V^{-}$ and $-p_{1,B}(0,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$, so she chooses to sell.

The uninformed speculator that short sells firm A and goes long on firm B: If $Q_{1,A} \in \{-2,-1\}$ or $Q_{1,B} \in \{1,2\}$, all actions generate zero profits, so she is indifferent. Suppose that $Q_{1,A} = Q_{1,B} = 0$. Buying firm A generates $p_{1,A}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}\overline{V}$ on firm A, and $-p_{1,B}(0,0) + V^{+}$ on firm B. Not trading yields $p_{1,A}(0,0) - \frac{2}{3}\overline{V}$ and $-p_{1,B}(0,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$. Selling generates $p_{1,A}(0,0) - \frac{1}{3}\overline{V}$ and $-p_{1,B}(0,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$, so she sells firm A again.

Finally, the uninformed speculator that does not trade stock A and goes long on firm B: If $Q_{1,A} = -1$ or $Q_{1,B} \in \{1,2\}$, all actions generate zero profits, so she is indifferent. Suppose $Q_{1,B} = 0$. If $Q_{1,A} = 1$, buying firm A generates $\overline{V} - \frac{2}{3}V^{+} - \frac{1}{3}p_{2,A}(1,0,0)$ and $V^{+} - p_{1,B}(1,0)$. Not trading yields 0 on firm A, and $-p_{1,B}(1,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$ on firm B, and selling generates $\frac{1}{3}p_{2,A}(1,0,0) - \frac{1}{3}\overline{V}$ and $-p_{1,B}(1,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$, so she chooses to sell. If $Q_{1,A} = 0$, buying yields $\frac{2}{3}\overline{V} - \frac{2}{3}V^{+}$ and $-p_{1,B}(0,0) + V^{+}$. Not trading generates 0 on stock A, and $-p_{1,B}(0,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$ on stock B. Selling generates 0 on stock A, and $-p_{1,B}(0,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$ on stock B, so she strictly prefers to sell.

C. Trading strategies in $t = 1$: Along the equilibrium path, the negatively informed speculator makes an expected profit of $\frac{1}{9}\left[(p_{1,A}(0,0) + \frac{1}{3}\overline{V} - \frac{2}{3}V^{-}) + (\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p_{1,B}(0,0))\right]$ if she short sells firm A in $t = 1$, where the first and second terms in parenthesis correspond to the profit on stocks A and B, respectively. If she does not trade in $t = 1$, her total expect profits are $\frac{1}{9}\left[(\frac{1}{3}p_{2,A}(1,0,0) + \frac{1}{3}\overline{V} - \frac{2}{3}V^{-}) + (\frac{2}{3}V^{+} + \frac{4}{3}V^{++} - p_{1,B}(0,0) - p_{1,B}(1,0))\right]$. The uninformed speculator makes an expected profit of $\frac{1}{9}\left[(p_{1,A}(0,0) - \frac{1}{3}\overline{V}) + (\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p_{1,B}(0,0))\right]$ if she short sells firm A in $t = 1$, and $\frac{1}{9}\left[(\frac{1}{3}p_{2,A}(1,0,0) - \frac{1}{3}\overline{V}) + (\frac{2}{3}V^{+} + \frac{4}{3}V^{++} - p_{1,B}(0,0) - p_{1,B}(1,0))\right]$ if she does not trade. For both speculators, the expression that defines the $t = 1$ trading strategy with respect to selling or not trading A is

$$p_{1,A}(0,0) - \frac{1}{3}p_{2,A}(1,0,0) \lesseqgtr \frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p_{1,B}(1,0). \tag{16}$$

Specifically, if the left-hand side of (16) is strictly greater than the right-hand side, both speculators short sell firm A in $t = 1$ with certainty. If (16) holds in equality, the speculators are indifferent and mix between the two. Finally, if the left-hand side of (16) is strictly less than the right-hand side, both speculators do not trade firm A in $t = 1$. Using stock prices from part A in (16) allows us to pin down μ in closed-form as reported in (6).

By following $\tilde{u}$, total *ex-ante* expected payoffs are as follows. The negatively informed and uninformed speculators make profits as described above. The positively informed speculator makes a profit of

$$\frac{1}{9}\left[\left(\frac{8}{3}V^{+} - p_{1,A}(1,0) - p_{1,A}(0,0) - \frac{1}{3}p_{2,A}(1,0,0) - \frac{1}{3}\overline{V}\right) + \left(p_{1,B}(1,0) + p_{1,B}(0,0) - 2V^{+}\right)\right]. \tag{17}$$

By assumptions (3)—(5), these are all strictly positive. Next, we show that there are not profitable deviations from the $\tilde{u}$ in $t = 1$.

Suppose that the positively informed speculator does not trade firm A in $t = 1$ while she continues to sell firm B in $t = 1$. If $Q_{1,A} = -1$ or $Q_{1,B} \in \{-2, -1\}$, all trades yield zero profits so the speculator is indifferent. Suppose that $Q_{1,B} = 0$. If $Q_{1,A} = 1$ and the speculator buys firm A in $t = 2$, her profit is $\frac{1}{3}V^{+} - p_{2,A}(1,0,0)$ on firm A and $p_{1,B}(1,0) - V^{+}$ on firm B. If she does not trade, she makes 0 profits on firm A, and $p_{1,B}(1,0) - \frac{2}{3}V^{+} - \frac{1}{3}V^{++}$ on firm B. Selling yields $\frac{1}{3}p_{2,A}(1,0,0) - \frac{1}{3}V^{+}$ and $p_{1,B}(1,0) - \frac{1}{3}V^{+} - \frac{2}{3}V^{++}$. Therefore, she will unambiguously choose to buy in $t = 2$. If $Q_{1,A} = 0$, buying generates a profit of $\frac{1}{3}V^{+} - \frac{1}{3}\overline{V}$ and $p_{1,B}(0,0) - V^{+}$. Not trading yields $p_{1,B}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}V^{++}$, and selling generates $\frac{1}{3}\overline{V} - \frac{1}{3}V^{+}$ and $p_{1,B}(1,0) - \frac{2}{3}V^{+} - \frac{1}{3}V^{++}$. Therefore, she chooses to buy. This deviation generates an *ex-ante* expected total profit of

$$\frac{1}{9}\left[\left(\frac{2}{3}V^{+} - \frac{1}{3}p_{2,A}(1,0,0) - \frac{1}{3}\overline{V}\right) + \left(p_{1,B}(1,0) + p_{1,B}(0,0) - 2V^{+}\right)\right],$$

which is smaller than (17) given that $2V^{+} > p_{1,A}(1,0) + p_{1,A}(0,0)$.

Now, suppose that the positively informed speculator does not trade firm A in $t = 1$ and goes long on firm B in $t = 1$. If $Q_{1,A} = -1$ or $Q_{1,B} \in \{1, 2\}$, all trades yield zero profits so the speculator is indifferent. Suppose that $Q_{1,B} = 0$. If $Q_{1,A} = 1$ and the speculator buys firm A in $t = 2$, her profit is $\frac{1}{3}V^{+} - \frac{1}{3}p_{2,A}(1,0,0)$ on firm A and $-p_{1,B}(1,0) + V^{+}$ on firm B. Not trading generates a profit of $-p_{1,B}(1,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$ on stock B only. Selling generates $\frac{1}{3}p_{2,A}(1,0,0) - \frac{1}{3}V^{+}$ and $-p_{1,B}(1,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$. Here, if $2V^{+} - p_{2,A}(1,0,0) > V^{++}$, the speculator chooses to buy. Otherwise, she sells. Now, suppose that $Q_{1,A} = 0$. If the speculator buys firm A in $t = 2$, her profit is $\frac{1}{3}V^{+} - \frac{1}{3}\overline{V}$ on firm A, and $-p_{1,B}(0,0) + V^{+}$ on firm B. Not trading generates a profit of $-p_{1,B}(0,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$, and selling yields $\frac{1}{3}\overline{V} - \frac{1}{3}V^{+}$ and

$-p_{1,B}(0,0)+\frac{1}{3}V^{+}+\frac{2}{3}V^{++}$. Here, if $2V^{+}-\overline{V}>V^{++}$, the speculator chooses to buy. Otherwise, she sells. Since $p_{2,A}(1,0,0)\geq\overline{V}$, if the speculator chooses to buy when $Q_{1,A}=1$, she will also buy when $Q_{1,A}=0$. Therefore, there are three cases to consider: (*i*) the speculator buys regardless of $Q_{1,A}$, (*ii*) the speculator sells when $Q_{1,A}=1$ and buys when $Q_{1,A}=0$, or (*iii*) she sells regardless of $Q_{1,A}$. We know that (*i*) cannot be optimal, because if the speculator were to not trade firm A in $t=1$ and buy in $t=2$, her optimal strategy in firm B would be to sell in $t=1$, and we already showed that this is dominated by the equilibrium strategy. In addition, (*ii*) is a combination of (*i*) and (*iii*) that does not pick the best position in both firms simultaneously.[8] Therefore, it suffices to show that (*iii*) is dominated by the equilibrium strategy of the positively informed speculator. This is true if

$$\frac{2}{3}V^{++}-p_{1,A}(1,0)-p_{1,A}(0,0)-\frac{2}{3}p_{2,A}(1,0,0)-\frac{2}{3}\overline{V}+2p_{1,B}(0,0)+2p_{1,B}(1,0)-\frac{4}{3}V^{++}>0.$$

Plugging $p_{1,A}(0,0)$ and $p_{1,B}(0,0)$, and using the fact that $2\overline{V}<V^{+}$. It suffices to show that

$$\frac{5+2\alpha}{6}V^{+}-p_{1,A}(1,0)-\frac{2}{3}p_{2,A}(1,0,0)+2p_{1,B}(1,0)-\frac{2\alpha}{3}V^{++}>0.$$

Moreover, since $2V^{+}>V^{++}$, and V^{+} is no less than $p_{1,A}(1,0)$ and $p_{2,A}(1,0,0)$, the above inequality is satisfied if

$$p_{1,B}(1,0)>\frac{5+6\alpha}{12}V^{+},$$

which holds, since $\frac{5+6\alpha}{12}<1$ for $\alpha\leq 1$. Therefore, a positively informed speculator does not deviate to not trading firm A in $t=1$.

Now, consider a positively informed speculator that sells firm A in $t=1$. Suppose that she buys firm B in $t=1$. If $Q_{1,A}\in\{-2,-1\}$ or $Q_{1,B}\in\{1,2\}$, all actions generate zero profits so she is indifferent. If $Q_{1,A}=Q_{1,B}=0$ and the speculator buys firm A in $t=1$, she makes a profit of $p_{1,A}(0,0)-\frac{2}{3}V^{+}-\frac{1}{3}\overline{V}$ on firm A, and $-p_{1,B}(0,0)+V^{+}$ on firm B. If she does not trade, her profits are $p_{1,A}(0,0)-\frac{2}{3}V^{+}$ and $-p_{1,B}(0,0)+\frac{2}{3}V^{+}+\frac{1}{3}V^{++}$. Selling generates $p_{1,A}(0,0)+\frac{1}{3}\overline{V}-\frac{2}{3}V^{+}$ and $-p_{1,B}(0,0)+\frac{1}{3}V^{+}+\frac{2}{3}V^{++}$, so she chooses to sell. The positively

[8] By selling when $Q_{1,A}=1$, the speculator makes a profit on firm B, but not on firm A. Similarly, by buying when $Q_{1,A}=0$, the speculator makes a profit on firm A, but not on firm B.

informed speculator will not deviate to this strategy if the following holds:

$$V^{+} - p_{1,A}(1,0) - 2p_{1,A}(0,0) - \frac{1}{3}p_{2,A}(1,0,0) - \frac{2}{3}\overline{V} + 2p_{1,B}(0,0) + p_{1,B}(1,0) - \frac{2}{3}V^{++} > 0.$$

Plugging $p_{1,A}(0,0)$ and $p_{1,B}(0,0)$, this expression becomes:

$$\frac{5}{3}V^{+} - p_{1,A}(1,0) - \frac{1}{3}p_{2,A}(1,0,0) - \frac{4}{3}\overline{V} + p_{1,B}(1,0) + \frac{2-2\alpha}{3}V^{++} > 0.$$

Since $V^{+} > 2\overline{V}$, it suffices to show that

$$V^{+} - p_{1,A}(1,0) - \frac{1}{3}p_{2,A}(1,0,0) + p_{1,B}(1,0) + \frac{2-2\alpha}{3}V^{++} > 0,$$

which holds, since both V^{+} and $p_{1,B}(1,0)$ are no less than $p_{1,A}(1,0)$ and $p_{2,A}(1,0,0)$, and $\alpha \leq 1$. Now, suppose that the positively informed speculator sells both firms in $t = 1$. If $Q_{1,A} \in \{-2,-1\}$ or $Q_{1,B} \in \{-2,-1\}$, all trades will generate zero profits regardless of whether firm A invests or not, so she is indifferent. If $Q_{1,A} = Q_{1,B} = 0$, buying firm A in $t = 2$ yields a profit of $p_{1,A}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}\overline{V}$ on firm A, and $p_{1,B}(0,0) - V^{+}$ on firm B. Not trading yields $p_{1,A}(0,0) - \frac{2}{3}V^{+}$ and $p_{1,B}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}V^{++}$. Selling generates $p_{1,A}(0,0) + \frac{1}{3}\overline{V} - \frac{2}{3}V^{+}$ and $p_{1,B}(0,0) - \frac{1}{3}V^{+} - \frac{2}{3}V^{++}$. If $V^{++} > V^{+} + \overline{V}$, she chooses to buy. Otherwise, she chooses to sell. However, if she were to sell firm A in $t = 2$ after selling it in $t = 1$, it would be unambiguously better to buy firm B in $t = 1$, which we already showed cannot be profitable. Therefore, assume that the speculator buys firm A in $t = 2$. In this case, her profit is

$$\frac{1}{9}\left[\left(p_{1,A}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}\overline{V}\right) + \left(p_{1,B}(0,0) - V^{+}\right)\right].$$

Plugging $p_{1,A}(0,0)$ and $p_{1,B}(0,0)$ shows that the above expression is negative by condition (3). Hence, we conclude that the positively speculator does not deviate from the equilibrium strategy $\tilde{u}$ in $t = 1$.

Next, consider a negatively informed speculator that buys firm A in $t = 1$. Suppose that she short sells firm B. If $Q_{1,A} = 2$ or $Q_{1,B} \in \{-2,-1\}$, all trades of stock B will generate zero profits, and any trade of firm A in $t = 2$ will generate at most a total profit of 0 when positions are liquidated. Suppose that $Q_{1,B} = 0$. If $Q_{1,A} = 1$ and the speculator buys firm A

again, her profit is $-p_{1,A}(1,0) - \frac{2}{3}V^{+} - \frac{1}{3}p_{2,A}(1,0,0) + 2V^{-}$ on firm A, and $p_{1,B}(1,0) - V^{+}$ on firm B. Not trading yields $-p_{1,A}(1,0) + \frac{2}{3}V^{-}$ and $p_{1,B}(1,0) - \frac{2}{3}V^{+} - \frac{1}{3}V^{++}$. Selling generates $-p_{1,A}(1,0) + \frac{1}{3}p_{2,A}(1,0,0)$ and $p_{1,B}(1,0) - \frac{1}{3}V^{+} - \frac{2}{3}V^{++}$. Selling yields negative profits on both firms, and not trading generates higher profits than buying, so the speculator chooses to not trade. If $Q_{1,A} = 0$, buying generates $-p_{1,A}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}\overline{V} + 2V^{-}$ on firm A, and $p_{1,B}(0,0) - V^{+}$ on firm B. Not trading generates $-p_{1,A}(0,0) + \frac{2}{3}V^{-}$ and $p_{1,B}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}V^{++}$. Selling generates $-p_{1,A}(0,0) + \frac{1}{3}\overline{V}$ and $p_{1,B}(0,0) - \frac{1}{3}V^{+} - \frac{2}{3}V^{++}$. Again, the speculator chooses to not trade. This deviation yields a total *ex-ante* profit of

$$\frac{1}{9}\left[\left(-p_{1,A}(0,0) - p_{1,A}(1,0) + \frac{4}{3}V^{-}\right) + \left(p_{1,B}(0,0) + p_{1,B}(1,0) - \frac{4}{3}V^{+} - \frac{2}{3}V^{++}\right)\right],$$

which is smaller than the equilibrium payoff of the negatively informed speculator if

$$\overline{V} + p_{1,A}(1,0) - 2V^{-} + V^{+} + \frac{2\alpha}{3}V^{++} - p_{1,B}(1,0) > 0.$$

Since $p_{1,A}(1,0) \geq \overline{V}$, it suffices to show that:

$$2V^{+} - V^{-} + \frac{2\alpha}{3}V^{++} - p_{1,B}(1,0) > 0,$$

which holds, because $2V^{+} > V^{++} \geq p_{1,B}(1,0)$ and $V^{-} < 0$. Now, suppose that the negatively informed speculator buys both firms in $t = 1$. If $Q_{1,A} = 2$ or $Q_{1,B} \in \{1, 2\}$, all trades generate at most zero profits regardless of whether firm A invests or not, so she is indifferent. Suppose that $Q_{1,B} = 0$. If $Q_{1,A} = 1$ and the speculator buys firm A again, her profit is $-p_{1,A}(1,0) - \frac{2}{3}V^{+} - \frac{1}{3}p_{2,A}(1,0,0) + 2V^{-}$ on firm A, and $-p_{1,B}(1,0) + V^{+}$ on firm B. If she does not trade, she makes a profit of $-p_{1,A}(0,0) + \frac{2}{3}V^{-}$ and $-p_{1,B}(0,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$. Selling generates $-p_{1,A}(1,0) + \frac{1}{3}p_{2,A}(1,0,0)$ and $-p_{1,B}(1,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$, so the speculator chooses to sell. If $Q_{1,A} = 0$, buying again generates $-p_{1,A}(0,0) - \frac{2}{3}V^{+} - \frac{1}{3}\overline{V} + 2V^{-}$ on firm A, and $-p_{1,B}(0,0) + V^{+}$ on firm B. Not trading yields $-p_{1,A}(0,0) + \frac{2}{3}V^{-}$ and $-p_{1,B}(0,0) + \frac{2}{3}V^{+} + \frac{1}{3}V^{++}$. Selling generates $-p_{1,A}(0,0) + \frac{1}{3}\overline{V}$ and $-p_{1,B}(0,0) + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$. Again, the speculator chooses to sell. This deviation yields a total *ex-ante* profit of

$$\frac{1}{9}\Bigg[\bigg(-p_{1,A}(1,0)-p_{1,A}(0,0)+\frac{1}{3}p_{2,A}(1,0,0)+\frac{1}{3}\overline{V}\bigg)+\\ \bigg(-p_{1,B}(0,0)-p_{1,B}(1,0)+\frac{2}{3}V^{+}+\frac{4}{3}V^{++}\bigg)\Bigg],$$

which is smaller than the negatively informed speculator's equilibrium profit if

$$2p_{1,A}(0,0)+p_{1,A}(1,0)-\frac{1}{3}p_{2,A}(1,0,0)-\frac{2}{3}V^{-}+p_{1,B}(0,0)-\frac{1}{3}V^{+}-\frac{2}{3}V^{++}>0.$$

Since $p_{1,A}(1,0)\geq\dfrac{1}{3}p_{2,A}(1,0,0)$ and $V^{-}<0$, it suffices to show that

$$2p_{1,A}(0,0)+p_{1,B}(0,0)-\frac{1}{3}V^{+}-\frac{2}{3}V^{++}>0.$$

Plugging prices, we can rewrite this condition as:

$$\frac{2}{3}\overline{V}+\alpha V^{+}-\frac{\alpha}{3}V^{++}>0,$$

which holds, by condition (3) and $\overline{V}>0$. Therefore, we conclude that the negatively informed speculator does not deviate from $\tilde{u}$.

Finally, consider an uninformed speculator that buys firm A in $t=1$. In addition, suppose that she sells firm B in $t=1$. If $Q_{1,A}=2$ or $Q_{1,B}\in\{-2,-1\}$, all trades of stock B will generate zero profits, and any trade of stock A in $t=2$ will generate at most a total profit of 0 when positions are liquidated. Suppose that $Q_{1,B}=0$. If $Q_{1,A}=1$, buying firm A again in $t=2$ generates a profit of $-p_{1,A}(1,0)-\frac{2}{3}V^{+}-\frac{1}{3}p_{2,A}(1,0,0)+2\overline{V}$ on firm A, and $p_{1,B}-V^{+}$ on firm B. Not trading yields $-p_{1,A}(1,0)+\frac{2}{3}\overline{V}$ and $p_{1,B}(1,0)-\frac{2}{3}V^{+}-\frac{1}{3}V^{++}$. Selling yields $-p_{1,A}(1,0)+\frac{1}{3}p_{2,A}(1,0,0)$ and $p_{1,B}(1,0)-\frac{1}{3}V^{+}-\frac{2}{3}V^{++}$. Selling yields strictly negative profits on both positions. However, not trading or buying do not necessarily generate negative profits, and it is ambiguous which one is larger. If $Q_{1,A}=0$, buying firm A again in $t=2$ generates a profit of $-p_{1,A}(0,0)-\frac{2}{3}V^{+}+\frac{5}{3}\overline{V}$ on firm A and $p_{1,B}(0,0)-V^{+}$ on firm B. Not trading generates $-p_{1,A}(0,0)+\frac{2}{3}\overline{V}$ and $p_{1,B}(0,0)-\frac{2}{3}V^{+}-\frac{1}{3}V^{++}$. Selling generates $-p_{1,A}(0,0)+\frac{1}{3}\overline{V}$ and $p_{1,B}(0,0)-\frac{1}{3}V^{+}-\frac{2}{3}V^{++}$. Again, selling generates negative profits, but not trading and selling are possibly positive, and their comparison depends on parameters. Therefore, we have to consider all possible combinations. In any case, if $Q_{1,A} = 2$ or $Q_{1,B} \in \{-2, -1\}$, the speculator sells firm A in $t = 2$ and make zero profits. Now, suppose that the speculator buys firm A in $t = 1$ if $Q_{1,A} = 1$ or $Q_{1,A} = 0$. In this case, her total *ex-ante* expected profit is

$$\frac{1}{9}\Big[\Big(-p_{1,A}(0,0) - p_{1,A}(1,0) - \frac{1}{3}p_{2,A}(1,0,0) - \frac{4}{3}V^{+} + \frac{11}{3}\overline{V}\Big) + \big(p_{1,B}(0,0) + p_{1,B}(1,0) - 2V^{+}\big)\Big]. \tag{18}$$

Now, let's consider two possibilities. First, suppose that $\mu = 0$, which implies that $p_{1,A}(1,0) = p_{2,A}(1,1,0) = p_{1,B}(1,0) = V^{+}$. In this case, the above expression will be negative if

$$-p_{1,A}(0,0) - \frac{11}{3}V^{+} + \frac{11}{3}\overline{V} + p_{1,B}(0,0) < 0.$$

Plugging the respective prices in the expression above, we can rewrite is as

$$5V^{+} > 5V^{-} + (2 - \alpha)V^{++},$$

which holds, by condition (3) and $V^{-} < 0$. Second, if $\mu > 0$, then (16) holds in equality. In this case, we can rewrite (18) as

$$\frac{1}{9}\Big[-2p_{1,A}(0,0) - p_{1,A}(1,0) - 3V^{+} + \frac{11}{3}\overline{V} + p_{1,B}(0,0) + \frac{2}{3}V^{++}\Big].$$

Since $p_{1,A}(1,0) \geq \overline{V}$, to show that the above expression is negative, it suffices to show that

$$-2p_{1,A}(0,0) - 3V^{+} + 3\overline{V} + p_{1,B}(0,0) + \frac{2}{3}V^{++} < 0.$$

Substituting prices on the above expression, we can rewrite it as condition (3). Therefore, the positively informed speculator will not buy firm A in $t = 2$. Now, suppose that the speculator does not trade firm A in $t = 1$ if $Q_{1,A} = 1$ or $Q_{1,A} = 0$. In this case, her total *ex-ante* expected profit is

$$\frac{1}{9}\Big[\Big(-p_{1,A}(0,0) - p_{1,A}(1,0) + \frac{4}{3}\overline{V}\Big) + \Big(p_{1,B}(1,0) + p_{1,B}(0,0) - \frac{4}{3}V^{+} - \frac{2}{3}V^{++}\Big)\Big]. \tag{19}$$

Again, first consider that $\mu = 0$. In this case, the above expression is negative if

$$-p_{1,A}(0,0) + \frac{4}{3}\overline{V} + p_{1,B}(0,0) - \frac{4}{3}V^{+} - \frac{2}{3}V^{++} < 0.$$

Using the prices in part A, the above expression is negative if

$$\overline{V} - V^{+} - \frac{\alpha}{3}V^{++} < 0,$$

which holds, by condition (3) and $V^{-} < 0$. Now, suppose that $\mu > 0$, in which case we can rewrite (19) as

$$\frac{1}{9}\left[-2p_{1,A}(0,0) - p_{1,A}(1,0) + \frac{1}{3}p_{2,A}(1,0,0) + \frac{4}{3}\overline{V} + p_{1,B}(0,0) - V^{+}\right].$$

Since $p_{1,A}(1,0)$ and $p_{2,A}(1,0,0)$ are both weakly greater than $\overline{V}$ and since we can write the equilibrium profit of the uninformed speculator as $\frac{\alpha}{3}V^{++}$, the speculator will not deviate if

$$-2p_{1,A}(0,0) + \frac{2}{3}\overline{V} + p_{1,B}(0,0) - V^{+} - \frac{\alpha}{3}V^{++} < 0.$$

Using prices from section A, this holds as long as $(2+\alpha)V^{+} > 2(1-\alpha)V^{++}$. In particular, if $\alpha \geq \frac{2}{5}$, this condition is implied by (3). Thus, we conclude that, if α is large enough, the uninformed speculator will not deviate to a null order on firm A in $t = 2$. Next, consider an uninformed speculator that buys firm A when $Q_{1,A} = 1$ and does not trade when $Q_{1,A} = 0$. In this case, her total *ex-ante* expected profit is

$$\begin{aligned}\frac{1}{9}\Bigg[&\left(-p_{1,A}(0,0) - p_{1,A}(1,0) - \frac{1}{3}p_{2,A}(1,0,0) + \frac{8}{3}\overline{V} - \frac{2}{3}V^{+}\right) + \\ &\left(p_{1,B}(0,0) + p_{1,B}(1,0) - \frac{5}{3}V^{+} - \frac{1}{3}V^{++}\right)\Bigg].\end{aligned} \tag{20}$$

If $\mu = 0$, the expression above is negative if

$$7V^{-} - 7V^{+} + 2(1-\alpha)V^{++} < 0,$$

which holds by (3) and $V^{-} < 0$. If $\mu > 0$ and considering that $p_{1,A}(1,0) \geq \overline{V}$, the speculator will not deviate if

$$-2p_{1,A}(0,0) - 2V^{+} + \frac{5}{3}\overline{V} + p_{1,B}(0,0) + \frac{1-\alpha}{3}V^{++} < 0. \tag{21}$$

Using prices from section A, the above expression holds as long as $(7+2\alpha)V^{+} > 3V^{-} + 2(3-2\alpha)V^{++}$, which is implied by (3) for $\alpha \geq \frac{1}{2}$. Next, consider an uninformed speculator that does not trade firm A when $Q_{1,A} = 1$ and buys when $Q_{1,A} = 1$. In this case, her total *ex-ante* expected profit is

$$\frac{1}{9}\Bigg[\left(-p_{1,A}(0,0) - p_{1,A}(1,0) + \frac{7}{3}\overline{V} - \frac{2}{3}V^{+}\right) + \left(p_{1,B}(0,0) + p_{1,B}(1,0) - \frac{5}{3}V^{+} - \frac{1}{3}V^{++}\right)\Bigg]. \tag{22}$$

If $\mu = 0$, the expression above is negative if

$$V^{-} - V^{+} + \frac{(1-\alpha)}{3}V^{++} < 0,$$

which, again, holds by (3) and $V^{-} < 0$. If $\mu > 0$, the speculator would not deviate if (21) holds, for which we already established the conditions. Therefore, we conclude that, if $\mu = 0$ or if $\alpha \geq \frac{1}{2}$, the uninformed speculator will not deviate to buying firm A in $t = 1$.

Now, suppose that the uninformed speculator buys both firms in $t = 1$. If $Q_{1,A} = 2$ or $Q_{2,A} \in \{1,2\}$, any trade will generate zero profits regardless of whether firm A invests or not. Suppose that $Q_{2,A} = 0$. If $Q_{1,A} = 1$, buying firm A again in $t = 2$ will generate a profit of $-p_{1,A}(1,0) - \frac{2}{3}V^{+} - \frac{1}{3}p_{2,A}(1,0,0) + 2\overline{V}$ on stock A, and $V^{+} - p_{1,B}(1,0)$ on stock B. Not trading yields $-p_{1,A}(1,0) + \frac{2}{3}\overline{V}$ and $\frac{2}{3}V^{+} + \frac{1}{3}V^{++} - p_{1,B}(1,0)$, and selling generates $-p_{1,A}(1,0) + \frac{1}{3}p_{2,A}(1,0,0)$ and $\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p_{1,B}(1,0)$. Clearly, the speculator chooses to sell. If $Q_{1,A} = 0$, buying firm A again in $t = 2$ generates a profit of $-p_{1,A}(0,0) - \frac{2}{3}V^{+} + \frac{5}{3}\overline{V}$ on firm A and $V^{+} - p_{1,B}(0,0)$ on firm B. Not trading generates $-p_{1,A}(0,0) + \frac{2}{3}\overline{V}$ and $\frac{2}{3}V^{+} + \frac{1}{3}V^{++} - p_{1,B}(0,0)$. Selling generates $-p_{1,A}(0,0) + \frac{1}{3}\overline{V}$ and $\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p_{1,B}(0,0)$, so she chooses to sell. This generates a total *ex-ante* expected profit of

$$\frac{1}{9}\Bigg[\left(-p_{1,A}(0,0) - p_{1,A}(1,0) + \frac{1}{3}p_{2,A}(1,0,0) + \frac{1}{3}\overline{V}\right) + \\ \left(\frac{2}{3}V^{+} + \frac{4}{3}V^{++} - p_{1,B}(0,0) - p_{1,B}(1,0)\right)\Bigg].$$

If $\mu = 0$, and using prices from part A, this expression is negative if $(4 + 2\alpha)V^{+} > (2 + \alpha)V^{++}$, which coincides with (3). If $\mu > 0$, the speculator will not deviate if

$$-p_{1,A}(1,0) + \frac{1}{3}\overline{V} - \frac{\alpha}{3}V^{+} < 0,$$

which holds, since $p_{1,A}(1,0) > \frac{1}{3}\overline{V}$. Therefore, we conclude that the uninformed speculator will not buy both stocks on $t = 1$.

Finally, the speculator does not deviate to not trading firm B in $t = 1$. The reason is intuitive. First, in equilibrium she always profits from her position in firm B in expectation. Second, by not trading firm B, she does not make $Q_{1,B} = 0$ any more likely, and $Q_{1,B} \neq 0$ does not generate positive profits on any position. Therefore, we conclude that the speculator does not deviate from $\tilde{u}$ overall.

D. Off-path beliefs: In our proposed equilibrium, the speculator mixes between all possible actions on her trading of firm A in $t = 2$ when $t = 1$ order flows reveals her position. As in Goldstein and Guembel (2008), this ensures that the equilibrium does not depend on beliefs about S given $Q_{2,A}$ when $Q_{1,A}$ or $Q_{1,B}$ reveals the speculator's position. In our model of two stocks, however, there are more possible off-paths outcomes that we need to address.

First, since the speculator cannot profit from $u_{2,B}$, we also assume that she mixes between all possible actions. Still, trading two stocks creates off-path order flows when the speculator deviates on her trading of one stock, but not the other. In particular, the order flows $(Q_{1,A}, Q_{1,B}) \in \{(2,2), (2,1), (-2,-2), (-1,-2), (-2,-1), (-1,-1)\}$ happen with probability zero along the equilibrium path. In these cases, it is not clear that firm A invests or not. For instance, $(2,2)$ reveals that the speculator went long on both firms, thus sending conflicting information on whether it should invest or not. If, in this case, firm A does not invest, then $p_{1,A} = 0$ and $p_{1,B} = V^{++}$ and firm B invests alone, in which case uncertainty is resolved and the speculator does not profit. On the other hand, if conflicting information leads firm A to invest for sure, then the feedback channel is effectively shut down. In both cases, $t = 2$ prices remain the same and the speculator randomizes across actions, so she does not profit on $t = 2$ either. A similar argument can be applied to the other off-path order flows. We conclude, therefore, that how we set investment decisions following off-the-equilibrium-path order flows does not affect our equilibrium structure. □

Proof of Proposition 2

The proof has three parts. In part A, we determine stock prices under trading strategy u^{sb} In parts B and C, we show that $t = 2$ and $t = 1$ trading strategies are optimal, respectively.

A. Stock prices and firms' investments: Again, firm B always invests, whereas firm A does not invest if it is revealed that the speculator is not positively informed. Under u^{sb}, this is reflected into $t = 2$ equilibrium stock prices as follows:

$$\begin{aligned}&p_{2,A}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = 0,\ p_{2,B}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = V^{++},\ \text{when } Q_{1,A} = -1 \text{ and}\\ &Q_{1,B} \in \{0, 1, 2\};\ \text{or when } Q_{1,A} \in \{0, 1\} \text{ and either } Q_{1,B} = 2 \text{ or } Q_{2,A} = -1,\end{aligned} \tag{23}$$

$$\begin{aligned}&p_{2,A}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = p_{2,B}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = V^{+},\ \text{when } Q_{1,A} = 2 \text{ and}\\ &Q_{1,B} \in \{-1, 0, 1\};\ \text{or when } Q_{1,A} \in \{0, 1\} \text{ and either } Q_{1,B} = -1 \text{ or } Q_{2,A} = 2,\end{aligned} \tag{24}$$

$$\begin{aligned}&p_{2,A}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = \overline{V} \text{ and } p_{2,B}(Q_{1,A}, Q_{2,A}, Q_{1,B}) = V^{+},\ \text{otherwise, when}\\ &Q_{1,A}, Q_{2,A}, Q_{1,B} \text{ do not reveal the speculator's trading.}\end{aligned} \tag{25}$$

Prices in $t = 1$ are given by

$$\begin{aligned}&p_{1,A}(Q_{1,A}, Q_{1,B}) = 0,\ p_{1,B}(Q_{1,A}, Q_{1,B}) = V^{++},\ \text{when } Q_{1,A} = -1 \text{ and}\\ &Q_{1,B} \in \{0, 1, 2\};\ \text{or when } Q_{1,A} \in \{0, 1\} \text{ and } Q_{1,B} = 2,\end{aligned} \tag{26}$$

$$\begin{aligned}&p_{1,A}(Q_{1,A}, Q_{1,B}) = p_{1,B}(Q_{1,A}, Q_{1,B}) = V^{+},\ \text{when } Q_{1,A} = 2 \text{ and } Q_{1,B} \in \{-1, 0, 1\};\\ &\text{or when } Q_{1,A} \in \{0, 1\} \text{ and } Q_{1,B} = -1,\end{aligned} \tag{27}$$

$$\begin{aligned}&p_{1,A}(Q_{1,A}, Q_{1,B}) = \frac{\alpha}{6}V^{+} + \frac{2}{3}\overline{V} \text{ and } p_{1,B}(Q_{1,A}, Q_{1,B}) = \frac{4+\alpha}{6}V^{+} + \frac{2-\alpha}{6}V^{++},\\ &\text{otherwise, when } Q_{1,A}, Q_{1,B} \text{ do not reveal the speculator's trading.}\end{aligned} \tag{28}$$

For ease of exposition, let the prices in (28) be $p^{nr}_{1,A}$ and $p^{nr}_{1,B}$.

B. Trading strategies in $t = 2$: Suppose that the speculator is positively informed and bought firm A in $t = 1$. If her position was revealed by the order flows, all actions generate zero profits so she is indifferent. If her position was not revealed, that is, $Q_{1,A} \in \{0, 1\}$ and $Q_{1,B} \in \{0, 1\}$ and she buys firm A again, her profit is $-p^{nr}_{1,A} - \frac{2}{3}\overline{V} + \frac{5}{3}V^{+}$. Not trading generates a profit of

$-p^{nr}_{1,A} + \frac{2}{3}V^+$ and selling generates $-p^{nr}_{1,A} + \frac{1}{3}\overline{V}$, so she buys.

The negatively informed speculator who did not trade firm A, bought firm B and was not revealed by the order flows: Buying firm A in $t = 2$ generates a profit of $-\frac{1}{3}V^+ - \frac{2}{3}\overline{V} + V^-$ on her position in firm A, and $-p^{nr}_{1,B} + V^+$ on firm B. Not trading firm A again generates $-p^{nr}_{1,B} + \frac{2}{3}V^+ + \frac{1}{3}V^{++}$ on firm B, so she does not trade.

The uninformed speculator who did not trade firm A, bought firm B and was not revealed by the order flows: Buying firm A in $t = 2$ generates a profit of $-\frac{1}{3}V^+ + \frac{1}{3}\overline{V}$ on firm A, and $-p^{nr}_{1,B} + V^+$ on firm B. Not trading firm A generates $-p^{nr}_{1,B} + \frac{2}{3}V^+ + \frac{1}{3}V^{++}$ on firm B, so she does not trade.

C. Trading strategies in $t = 1$: Regardless of the speculator's signal, the probability of her $t = 1$ trading not being revealed by $Q_{1,A}, Q_{1,B}$ is $\frac{4}{9}$. Therefore, the speculator's total *ex-ante* equilibrium profit is $\frac{4}{9}\left(\frac{5}{3}V^+ - p^{nr}_{1,A} - \frac{2}{3}\overline{V}\right)$ if positively informed, and $\frac{4}{9}\left(\frac{2}{3}V^+ + \frac{1}{3}V^{++} - p^{nr}_{1,B}\right)$ if either negatively informed or uninformed.

Suppose that the positively informed speculator does not trade firm A neither B in $t = 1$. If either $Q_{1,A} = -1$ or $Q_{1,B} = -1$, all trades in $t = 2$ generate zero profits so she is indifferent across actions. Otherwise, buying firm A in $t = 2$ yields a profit of $-\frac{2}{3}\overline{V} + \frac{2}{3}V^+$. Not trading firm A generates a profit of 0, so she buys. This deviation generates a total *ex-ante* profit of $\frac{4}{9}\left(\frac{2}{3}V^+ - \frac{2}{3}\overline{V}\right)$, which is lower than the equilibrium profit since $p^{nr}_{1,A} \leq V^+$, so the positively informed speculator does not deviate.

The positively informed speculator does not trade firm A and buys firm B: If either $Q_{1,A} = -1$ or $Q_{1,B} = 2$, all trades in $t = 2$ generate zero profits so she is indifferent across actions. Otherwise, buying firm A in $t = 2$ generates a profit of $-\frac{2}{3}\overline{V} + \frac{2}{3}V^+$ on firm A's position, and $-p^{nr}_{1,B} + V^+$ on firm B. This profit is lower than the one from not trading both firms in $t = 1$ and buying firm A in $t = 2$, which we already showed is smaller than the equilibrium profit. Not trading firm A in $t = 2$ yields $-p^{nr}_{1,B} + \frac{2}{3}V^+ + \frac{1}{3}V^{++}$. This deviation is not profitable as long as $V^+ \geq \frac{\alpha}{6}V^{++} + \frac{2}{3}V^-$, which holds, since $V^- < 0$ and by (3). Therefore, we conclude that the positively informed speculator does not deviate from u^{sb}.

Suppose that the negatively informed speculator buys both firms in $t = 1$. If either $Q_{1,A} = 2$ or $Q_{1,B} = 2$, all trades generate zero profits so she indifferent across actions. Otherwise, buying firm A again generates $-p^{nr}_{1,A} - \frac{1}{3}V^+ - \frac{2}{3}\overline{V} + 2V^-$ on firm A, and $-p^{nr}_{1,B} + V^+$ on firm B. Not trading firm A generates $-p^{nr}_{1,A} + \frac{2}{3}V^-$ on firm A, and $\frac{2}{3}V^+ + \frac{1}{3}V^{++} - p^{nr}_{1,B}$ on firm B. Selling firm A yields $-p^{nr}_{1,A} + \frac{1}{3}\overline{V}$ on firm A, and $\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p^{nr}_{1,B}$ on firm B, so she chooses to sell. This deviation generates a total ex-ante profit of

$$\frac{4}{9}\left[\left(\frac{1}{3}\overline{V} - p^{nr}_{1,A}\right) + \left(\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p^{nr}_{1,B}\right)\right],$$

which is smaller than the equilibrium profit by condition (7). Now, suppose that the negatively informed speculator buys firm A and does not trade firm B in $t = 1$. If either $Q_{1,A} = 2$ or $Q_{1,B} = -1$, all trades generate zero profits so she indifferent across actions. Otherwise, buying, not trading or selling firm A in $t = 2$ generates the same profit on firm A as the negatively speculator who buys both firms in $t = 1$ and no profits on stock B. All these actions generate unambiguously negative profits. Therefore, we conclude that the negatively informed speculator does not deviate from the equilibrium strategy.

Suppose that the uninformed speculator buys both firms in $t = 1$. If either $Q_{1,A} = 2$ or $Q_{1,B} = 2$, all trades generate zero profits so she indifferent across actions. Otherwise, buying firm A again generates $-p^{nr}_{1,A} - \frac{1}{3}V^{+} + \frac{4}{3}\overline{V}$ on firm A, and $-p^{nr}_{1,B} + V^{+}$ on firm B. Not trading firm A yields $-p^{nr}_{1,A} + \frac{2}{3}\overline{V}$ on firm A, and $\frac{2}{3}V^{+} + \frac{1}{3}V^{++} - p^{nr}_{1,B}$ on firm B. Selling generates $-p^{nr}_{1,A} + \frac{1}{3}\overline{V}$ on firm A, and $\frac{1}{3}V^{+} + \frac{2}{3}V^{++} - p^{nr}_{1,B}$ on firm B. Note that this is the same profit of the negatively informed speculator that plays this strategy. As the speculator does not hold firm A until investment decisions, the payoff ultimately does not depend on firm A's type. Here, the speculator will either not trade or sell, depending on the parameters. Since we already established that selling is dominated by the equilibrium strategy by (7), it suffices to show that following u^{sb} generates a higher profit than not trading in $t = 2$.[9] Not trading in $t = 2$ yields a total ex-ante profit of

$$\frac{4}{9}\left[\left(\frac{2}{3}\overline{V} - p^{nr}_{1,A}\right) + \left(\frac{2}{3}V^{+} + \frac{1}{3}V^{++} - p^{nr}_{1,B}\right)\right],$$

which is smaller than the equilibrium profit because $p^{nr}_{1,A} \geq \frac{2}{3}\overline{V}$.

Finally, suppose that the uninformed speculator buys firm A and does not trade firm B in $t = 1$. Conditional on her $t = 1$ trading not being revealed by order flows, her profits on firm A are the same as the uninformed speculator that buys both firms in $t = 1$ across all actions, which

[9] Recall that the negatively informed and uninformed speculators make the same profit in equilibrium.

are negative. Therefore, we conclude that the uninformed speculator does not deviate from u^{sb}, which concludes the proof. $\square$

Proof of Corollary 2

In our equilibrium with short selling in Proposition 1, firm A always invests when $S = \theta^l$, and invests with probability $\frac{1}{27}$ when $S \in \{\theta^h, \emptyset\}$ and the speculator short sells firm A in $t = 1$. Therefore, total *ex-ante* expected values of each firm are

$$\begin{aligned} \mathbb{E}(V_A^{ss}) &= \frac{\alpha}{2} V^+ + \frac{\alpha}{2} \frac{1}{27} V^- + (1-\alpha) \frac{1}{27} \overline{V}, \\ \mathbb{E}(V_B^{ss}) &= \frac{\alpha}{2} V^+ + \frac{2-\alpha}{2} \left(\frac{1}{27} V^+ + \frac{26}{27} V^{++} \right). \end{aligned} \tag{29}$$

When $S \in \{\theta^h, \emptyset\}$ and the speculator does not trade firm A in $t = 1$, expected values are

$$\begin{aligned} \mathbb{E}(V_A^{ss}) &= \frac{\alpha}{2} V^+ + \frac{\alpha}{2} \frac{2}{27} V^- + (1-\alpha) \frac{2}{27} \overline{V}, \\ \mathbb{E}(V_B^{ss}) &= \frac{\alpha}{2} V^+ + \frac{2-\alpha}{2} \left(\frac{2}{27} V^+ + \frac{25}{27} V^{++} \right). \end{aligned} \tag{30}$$

In our equilibrium under a short selling ban in Proposition 2, firm A also invests with certainty. When $S \in \{\theta^h, \emptyset\}$, firm A enters with probability $\frac{8}{27}$. Therefore, expected values are

$$\begin{aligned} \mathbb{E}(V_A^{sb}) &= \frac{\alpha}{2} V^+ + \frac{\alpha}{2} \frac{8}{27} V^- + (1-\alpha) \frac{8}{27} \overline{V}, \\ \mathbb{E}(V_B^{sb}) &= \frac{\alpha}{2} V^+ + \frac{2-\alpha}{2} \left(\frac{8}{27} V^+ + \frac{19}{27} V^{++} \right). \end{aligned} \tag{31}$$

Clearly, the expected value of firm B decreases under a short selling ban, since firm A is overall more likely to invest and $V^{++} > V^+$. As for firm A, the condition for its expected value in either (29) or (30) to be larger than in (31) are the same and coincide with assumption (5).

In addition, we have that

$$\frac{\partial}{\partial \alpha} \left(\mathbb{E}(V_A^{sb}) - \mathbb{E}(V_A^{ss}) \right) = \frac{7}{27} \frac{V^+}{2} > 0,$$

if the negatively informed and uninformed speculators short sell firm A in $t = 1$ when following $\tilde{u}$, with a similar expression for when they do not trade firm A in $t = 1$. In the case of firm B, we have that

$$\frac{\partial}{\partial \alpha}\left(\mathbb{E}(V_B^{sb}) - \mathbb{E}(V_B^{ss})\right) = \frac{1}{2}\frac{1}{27}\left(V^{+} - V^{++}\right) < 0,$$

if the negatively informed and uninformed speculators short sell firm A in $t = 1$ when following $\tilde{u}$, with a similar expression for when they do not trade firm A in $t = 1$. □

Proof of Proposition 3

When the negatively informed and uninformed speculators follow the strategy described in Proposition 3, the only instances where the speculator's trading is revealed in $t = 1$ is $Q_{1,B} \in \{-1, 2\}$. Otherwise, $t = 1$ prices are

$$p_{1,A}(Q_{1,A}, Q_{1,B}) = \frac{\alpha}{3}V^{+} + \frac{1}{3}\overline{V},\ p_{1,B}(Q_{1,A}, Q_{1,B}) = \frac{1+\alpha}{3}V^{+} + \frac{2-\alpha}{3}V^{++}. \tag{32}$$

When either the negatively informed on uninformed speculators follow the proposed strategy, their expected profit at $t = 2$ when they sell firm A and hold firm B is $-p_{1,A} + \frac{1}{3}\overline{V}$ on firm A, and $-p_{1,B} + \frac{1}{3}V^{+} + \frac{2}{3}V^{++}$ on firm B, where prices are given in (32). Thus, the total profit is $\frac{\alpha}{3}\left(V^{++} - 2V^{++}\right)$, which is negative by assumption (4). □